

8 Rudimentary Search Modelling

In the preceding chapters the processes of acquisition have been limited to the detection of objects in a known position in the visual field. Whilst such knowledge is a necessary foundation from which to develop an appreciation of the practical functions of vision, the visual task is frequently more complicated. If detection of the presence of an object somewhere in the visual field is the task one is concerned with – as, for instance, in the task of detecting the presence of an aircraft in the sky or of a vehicle in open country – then the important difference between this task and basic detection threshold studies discussed so far is the need to *search* for the object over an extended field. The search process is discussed at length by Koopman¹, Krendel and Wodinsky² and Bloomfield³, amongst others.

During a search process the eye scans the scene in a series of jumps, dwelling for a fraction of a second on each area. Such a series of jumps and dwells are the search equivalent of the involuntary saccades and inter-saccadic intervals described for fixated vision in earlier chapters. In an empty field situation – that is, where the background scene contains little structure – the jumps, or voluntary saccades, are usually considered to be random (e.g. Koopman¹ and Krendel and Wodinsky²). Conversely, for structured backgrounds, they are often found to be concentrated in areas of high interest, as might be expected (e.g. Enoch⁴ and Williams⁵). Each dwell period is conventionally referred to in search modelling as a *glimpse*. The glimpse frequency is usually of the same order as that for involuntary saccades – that is, about three per second on average – although there is some evidence that, for tasks which are *felt* to be difficult, this frequency drops to around one per second (e.g. Enoch⁴). At each and every glimpse there exists a probability either that the gaze will be centred on the object – yielding a foveal single glimpse viewing situation – or that the object may be seen anywhere within a larger or smaller region of peripheral vision as determined by the viewing conditions. As shown in Section 4.8, the retinal position on which the object falls on a particular glimpse will modify greatly the chances of seeing the object on that glimpse. Furthermore, it seems in practice that, if there is a reasonably high confidence of having seen an object in the peripheral field at a given glimpse, the next glimpse will be drawn towards that object, thus strengthening the effect of the stimulus and allowing confirmation or rejection as being the object of interest.

In this chapter we shall be concerned with the rudimentary search situation – the detection of an object in an unknown position in a plain field. Other forms of search – and in particular that of searching a structured field for a particular object amongst similar (or confusable) objects – will be dealt with in Section 13.3.

8.1 VISUAL LOBE CONCEPTS

In order to attempt to study empty field search it is first necessary to find some way of defining the relative probability of perceiving an object at various points in the visual field. This in turn requires that we combine, in some way, the trends of 50% thresholds as discussed in Section 4.8 with the frequency of seeing concepts discussed in Chapter 3. Such a combination process leads to a set of probability curves which are in some ways analogous to the illuminance cross-sections of the retinal images of a group of progressively smaller disc stimuli due to interactions of the spread function with the geometric shape^{6,7}. For instance, for small targets and suprathreshold foveal single glimpse situations the probability will remain at essentially 100% out to some retinal eccentricity, after which the probability will fall off progressively, much as the illuminance of a degraded image falls off. However, the shape of the function must be expected to be determined both by the size and shape of the object and by the retinal receptor density function derived from Østerberg (Section 2.4).

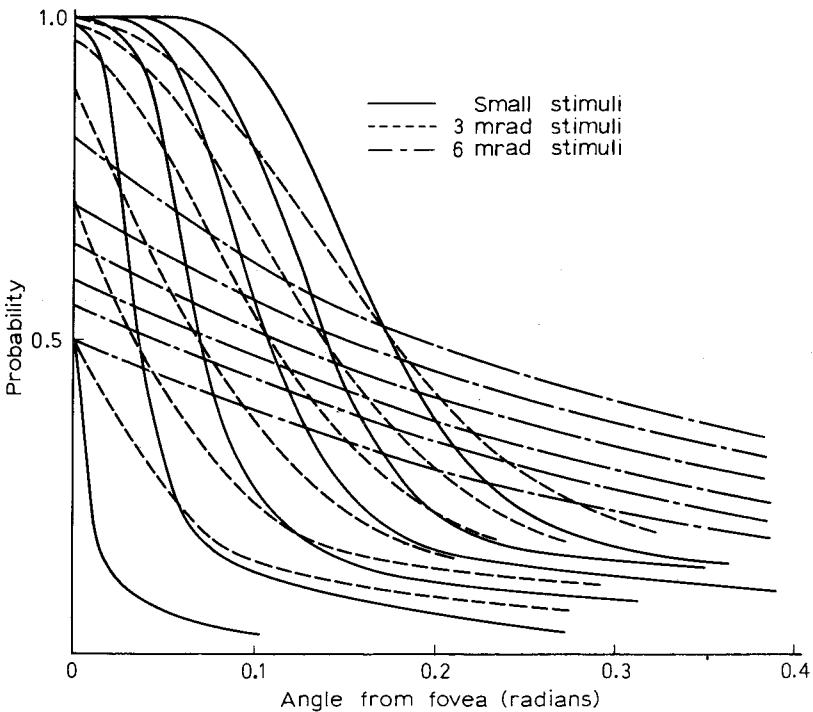


Fig. 8.1. Variation of single-glimpse probability p_G with retina l eccentricity as a function of stimulus contrast (groups of curves are shown for various visual lobe radii at $p_G = 0.5$).

The result is that, for large objects, the change of probability with retinal position is likely to be very small. This is in keeping with the peripheral threshold trends (see Section 4.8). As examples of the form of such probability maps, the peripheral probabilities computed from experimental threshold data and assumed frequency of seeing spreads for a series of contrasts are shown for large and small circular stimuli in Fig. 8.1. Such probability cross-sections are known as visual lobes.

The computation of visual lobes is relatively complicated. In an attempt to simplify practical usage of visual lobe concepts it has been proposed by a number of people (e.g. Owen⁸ and Davies^{9,10}) that it is adequate, for most purposes, to define each particular visual lobe as a cylindrical function with an angular diameter equal to the angle at which, for a given situation, there exists a 50% probability of single glimpse acquisition. Although it is difficult to provide positive proof of the universality of such an approximation, the idea enjoys wide acceptance. It is believed by the author, from certain sensitivity modelling, that such an approximation is frequently entirely adequate in practice, despite the soft visual lobe profiles illustrated in Fig. 8.1. For this reason, in order to retain some simplicity in the modelling developed in this book, we shall only consider this simplified, approximate visual lobe concept at any length. It should be noted by the reader at this point that it is common practice to refer to the rigorous visual lobe concept as 'soft-shelled', whilst the approximate form to be developed further is known as 'hard-shelled'.

8.2 SINGLE GLIMPSE PROBABILITY

In order to determine the total probability of seeing an object in a given time in terms of hard-shell visual lobe concepts, it is first necessary to determine the probability of seeing in a single glimpse. This in turn must obviously be dependent on the size of the field to be searched. It must also be dependent on the size of the visual lobe. Now the size of the visual lobe for images which are effectively static on the retina may be determined, for a given object size and contrast, by a reordering of Equation 7.18. From this equation one may write, for threshold,

$$\left(\frac{180\theta_v}{\pi} + 1 \right) = \frac{(K_2\epsilon - \delta)n^2}{K_1}$$

or

$$\theta_v = \left\{ \frac{(K_2\epsilon - \delta)n^2}{K_1} - 1 \right\} \frac{\pi}{180} \quad (8.1)$$

where θ_v is the radius of the hard shell visual lobe.

For images which are moving on the retina, although in principle a similar approach may be made, reference back to Equations 7.20 and 7.21 will show that the situation becomes very complex. Only when motion is very fast compared to stimulus size is the dynamic situation easy, and in such a situation Equation 7.21 implies a trend towards a condition where threshold is constant over the majority of the retina (i.e. an equal probability of seeing wherever the stimulus is presented). This latter statement should not, however, be taken as an absolute fact, but rather as an indication of the order of things.

Returning to the essentially static retinal image situation, it is necessary to determine the single glimpse probability for a given lobe size within a given field. Now the simple expression for the probability of seeing a target anywhere in a search area A_F , when the visual lobe area is a_v , is given as

$$p_G = \frac{a_v}{A_F} \tag{8.2}$$

However, the inadequacy of this expression is apparent from the fact that if $a_v > A_F$ then $p_G > 1$, which is not possible. The reason for this is, of course, that if $a_v > A_F$, a portion of the visual lobe of size $(a_v - A_F)$ is wasted outside the search area. A frequently used correction for this situation is to employ the expression

$$p_G = \frac{a_v}{(A_F + a_v)} \tag{8.3}$$

which has the correct limiting behaviour. However, it is shown by Davies⁹ that, if one develops an exact solution, this should have the form

$$p_G = \left[\sqrt{\left(\frac{a_v}{A_F}\right) - \frac{a_v}{4A_F}} \right]^2 \tag{8.4}$$

which reaches $p_G = 1$ at $a_v = 4A_F$. It should be noted that this function only applies for

$$0 < a_v \leq 4A_F.$$

For $a_v > 4A_F$, $p_G = 1$.

An alternative way of expressing Equation 8.4 is in terms of rectangular or elliptical functions, when probability is given as

$$p_G = \left(\frac{x_v}{X_F} - \frac{x_v^2}{4X_F^2} \right) \left(\frac{y_v}{Y_F} - \frac{y_v^2}{4Y_F^2} \right) \tag{8.5}$$

where x_v and y_v are the two dimensions of the visual lobe and X_F and Y_F are the two dimensions of the search field. In the particular case of assumed circular visual lobes and search fields, by symmetry and inspection, one may conclude

that

$$p_G = \left(\frac{\theta_v}{\theta_F} - \frac{\theta_v^2}{4\theta_F^2} \right)^2 \text{ for } \theta_v \leq 2\theta_F \quad (8.6)$$

and

$$p_G = 1 \text{ for } \theta_v > 2\theta_F$$

where θ_F is the radius of the search field.

The reader wishing to study the topic of single glimpse probability in depth is referred to the extended treatise by Davies⁹.

8.3 CUMULATIVE SEARCH PROBABILITY

Having established the single glimpse probability it is necessary to consider how the cumulative probability of seeing builds up with increasing search time. Here we find ourselves with a situation similar to that already invoked for build up of intelligence about the existence of a stimulus in the fixated viewing situation (Section 7.3). If the glimpses are truly independent (as they should be in searching an empty field), if the stimulus remains of constant strength, and if the foveal probability is unity, then the cumulative probability of detection in m glimpses may be defined (e.g. Koopman¹, Krendel and Wodinsky² and Weatherburn¹¹) as

$$\Phi_m = 1 - \Pi(1 - p_G) = 1 - (1 - p_G)^m \quad (8.7)$$

Assuming an average glimpse rate of 3 per second this yields a family of curves as shown in Fig. 8.2. In practice the accumulations with time, when extrapolated back to zero probability, are found to follow curves closely resembling these theoretical curves, but with small time delays before any significant probability builds up (e.g. Krendel and Wodinsky² and Bloomfield³). This is, of course, due to the fact that response, in a multiple glimpse situation, tends to occur *at the end* of a particular glimpse, thus adding a delay time of the order of 1/3 s to the theoretical curves.

The above is entirely adequate for a situation where the single glimpse probabilities are constant with time (i.e. the stimulus does not change with time) and the foveal probability is unity. However, what of the many situations where one or both these conditions do not hold?

Let us first consider the situation where p_G is constant, but the foveal single glimpse probability is less than unity. In such a situation it seems wrong to consider the single glimpse value of q^{11} to be equatable to $(1 - p_G)$, since this then takes no account of the foveal probability p_f . At the same time, although Φ_m for multiple glimpses may be larger than p_f , it does not necessarily reach unity if p_f is small.

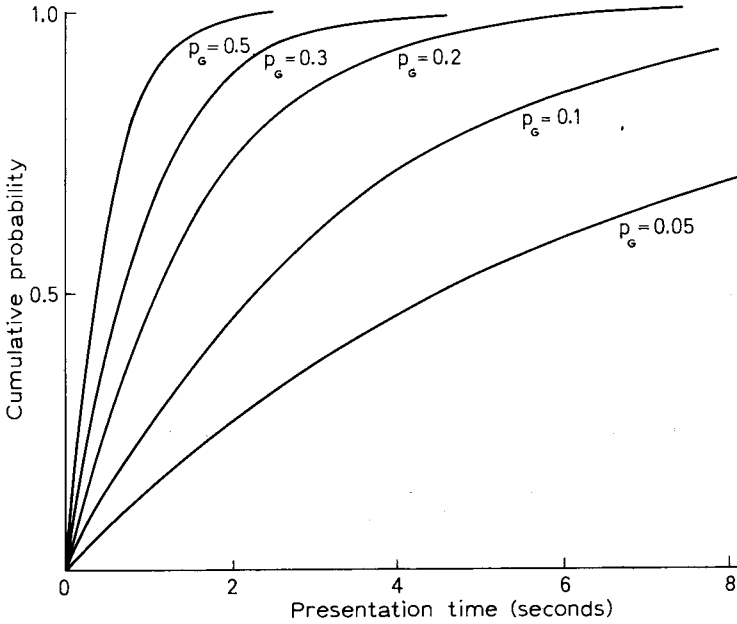


Fig. 8.2. Cumulative probability as a function of time for constant stimuli and various single glimpse probabilities (mean glimpse time $\frac{1}{3}$ s).

In an attempt to allow for these two factors, some workers have suggested that Equation 8.7 should be written

$$\begin{aligned} \Phi_m &= 1 - (1 - p_f p_G)^m, & m < M \\ \Phi_m &= \Phi_f, & m \geq M \end{aligned} \tag{8.8}$$

where M is the number of glimpses for the computed cumulative probability to reach the foveal probability for infinite viewing time Φ_f . Whilst such a function does provide for a reasonable value of

$$q = (1 - p_f p_G)$$

this being now related to a visual lobe size defined where probability is $p_f/2$, the discontinuity when Φ_m reaches Φ_f does not seem correct. Rather the present author believes that one should find a function which approaches the limit value of Φ_f asymptotically. Such a function would be arrived at by assuming a need to multiply all cumulative probabilities in an equation such as Equation 8.7 by the factor Φ_f , thus scaling the usual cumulative probability curve given by Equation 8.7 proportionately over its entire length. It is thus proposed that a suitable

function for Φ_m for any constant strength stimulus would be

$$\Phi_m = \Phi_f - \Phi_f(1 - p_f p_G)^m \quad (8.9)$$

Although Equation 8.9 allows computation of cumulative probabilities for many situations as a function of time (in units of number of glimpses m), it cannot be used as it stands if the stimulus strength is varying. Such is frequently the situation in, for instance, viewing of or from moving vehicles, aircraft, etc. In such situations the cumulative probability can only be found by a step by step computation using a rigorous form of Equation 8.9, viz.

$$\Phi_m = \Phi_{fm} - \Phi_{fm} \Pi(1 - p_{fr} p_{Gr}) \quad (8.10)$$

where Φ_{fm} is the foveal probability for infinite viewing time which exists after m glimpses, and p_{fr} and p_{Gr} are the foveal and general single glimpse probabilities on the r th glimpse. Fortunately, if the search field is large and the stimulus only small, the values of Φ_f and p_f will usually reach unity before p_G becomes significant (c.f. Fig. 8.1 and Equation 8.6). If these conditions are combined with a fairly rapid stimulus growth through threshold then the errors in Φ_m due to ignoring contributions where Φ_f and p_f are less than unity are very small and Equation 8.9 may be used. If, however, the stimulus growth is slow, and particularly for large stimuli and small search fields, it is believed that serious errors may arise due to such simplification.

8.4 PRACTICAL APPLICATIONS

In order to illustrate both the application and the type of results coming out of the rudimentary search modelling discussed in this chapter, let us consider some practical examples.

8.4.1 Search for a static stimulus

Such a situation is applicable to search for particular stars in astronomy, search for signal lights in a poor visibility situation and search for isolated threshold objects in such spheres as microscopy. Since the stimulus is essentially static, the value of p_G remains constant. Φ_m is thus a simple function of time. However, the accumulation is very dependent on the values of p_G and the foveal single glimpse probability p_f . These in turn are dependent on stimulus size, shape and contrast, whilst, in addition, p_G is dependent on search field radius θ_F .

Practical measurements of the search time as a function of object size, object contrast and search field diameter for circular stimuli were carried out by Krendel and Wodinsky^{1,2}. As an attempt to check the adequacy of the modelling of empty field search described in the previous section, a number of selected conditions from Krendel and Wodinsky's work were modelled by the present

author¹³. It was found that good predictions were obtained for a variety of target sizes and contrasts with large search fields, but that there was considerable disagreement with small search fields (of the order of 0.12 rad diameter). However, this disagreement at small search fields is in general agreement with the findings of Enoch¹⁴ that an increasing number of wasted glimpses occur during empty field search of fields less than 0.2 rad in diameter. It would thus appear that hard shell visual lobe search modelling is tentatively adequate in a practical static stimulus situation, provided that due account is taken of wasted glimpses for small search fields.

8.4.2 Search for a growing stimulus

An alternative and common search situation is that where the stimulus is growing in either size or contrast whilst search is progressing. In such a situation it is most likely that the cumulative probability is required not as a function of time but as either a direct or indirect function of the growth of the stimulus.

One particularly important practical case is in the acquisition of objects of interest from aircraft or of aircraft from the ground. In either of these situations the stimulus is growing rapidly in both size and contrast due to the rapid closure of viewing range. It is then most useful to compute Φ_m as a function of range. Such a computation may be accomplished by combining Equation 7.18 with Equation 8.10. Firstly, by defining ϵ and n in Equation 7.18 as functions of viewing range, a viewing range R is determined at which the foveal probability p_f is 50%. Then the viewing range is permitted to reduce by finite and convenient increments ΔR . For a median value of the first increment of reduction the hard shell visual lobe radius θ_v is computed from Equation 7.18. From this visual lobe and the effective search field radius θ_F , the value of p_G is computed (from Equation 8.6). A value of p_f is next computed by putting $\theta = 0$ in Equation 7.18. Knowing the rate of closure of range, the number of glimpses in this first range increment is determined. Then, from Equation 8.9, assuming constant probabilities in the small range increment, a value of Φ_m may be determined at a range $(R - \Delta R)$.

The above procedure may now be repeated for the next and subsequent increments of range reduction, except that at each subsequent stage the cumulative probability Φ_m includes not only the contribution $(1 - p_f p_G)_m$ for the increment but also the accumulation up to that point. In other words we may modify Equation 8.10 to read

$$\Phi_m = \Phi_f - \Phi_f(1 - p_f p_G)_m \times (\Phi'_f - \Phi'_m) / \Phi'_f \quad (8.11)$$

where $(\Phi'_f - \Phi'_m) / \Phi'_f$ refers to the accumulation up to the end of the previous range increment.

The above looks unpleasantly complicated with all the variable p_f 's and Φ_f 's to consider. Fortunately, as previously discussed, in many practical growth

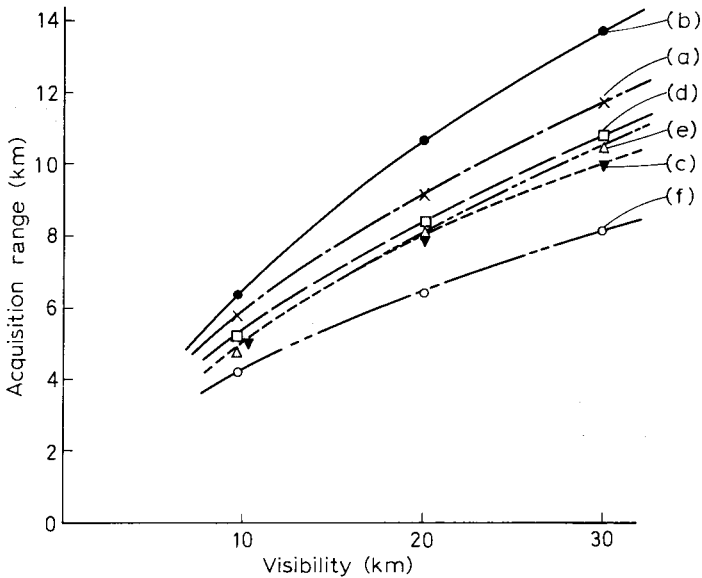


Fig. 8.3. Illustrating the prediction of variation of acquisition range with meteorological visibility for a target viewed through high power binoculars and approaching at 200 m/s. Curve (a): Reference conditions; $\theta_F = 0.435$ rad, Intrinsic contrast (C_O) = -0.9, Equivalent circle diameter in eye space (D_e) = 30 m, Field luminance $B' = 100$ cd/m² (approx). Single parameter variations from reference conditions:— Curve (b): $B' = 1\ 000$ cd/m² (approx). Curve (c): $C_O = -0.6$. Curve (d): $D_e = 23$ m. Curve (e): $\theta_F = 0.87$ rad. Twin parameter variation from reference condition (lower power binoculars):— Curve (f): $\theta_F = 0.11$ rad, $D_e = 7.2$ m. (Reproduced from Overington¹³ by courtesy of the Advisory Group for Aerospace Research and Development of NATO).

situations involving moderate search, almost all the accumulation will occur after p_f and Φ_f have reached unity. Thus, for many practical purposes, it is adequate to consider Equation 8.11 to be replaced by

$$\Phi_m = 1 - (1 - p_G)_m(1 - \Phi'_m) \quad (8.12)$$

where Φ'_m is the accumulation to the end of the previous range reduction increment.

Some examples of theoretical and practical growth functions are to be found in Davies^{9,10} and Overington¹³, whilst a few computed examples illustrating effects of θ_F and stimulus parameters are to be found in Figs. 8.3 and 8.4. In order to compute such functions it was imperative that allowance be made for atmospheric effects on contrast. The form and modelling of these latter effects are discussed in Chapter 15.

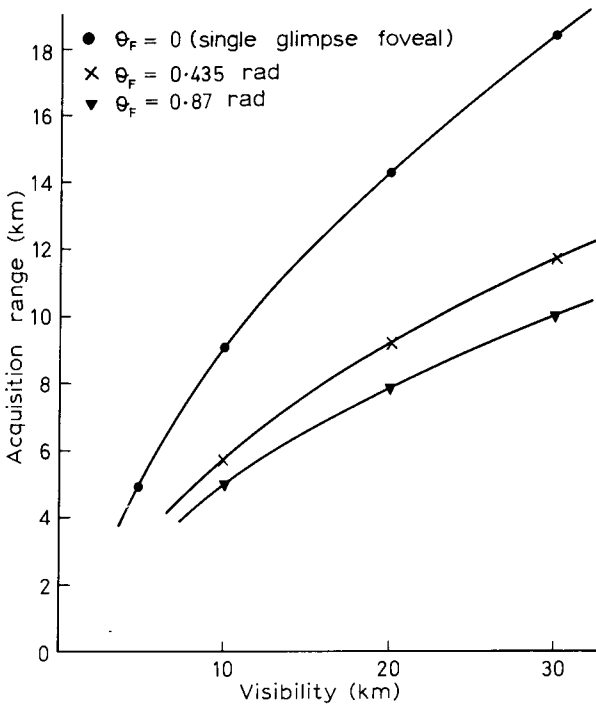


Fig. 8.4. Illustrating the sensitivity to search field size of the acquisition range/visibility function for a typical target approaching at 200 m/s and viewed through high power binoculars. (Reproduced from Overington^{1,3} by courtesy of the Advisory Group for Aerospace Research and Development of NATO).

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